Organisation of diffusion-driven stripe formation in expanding domains

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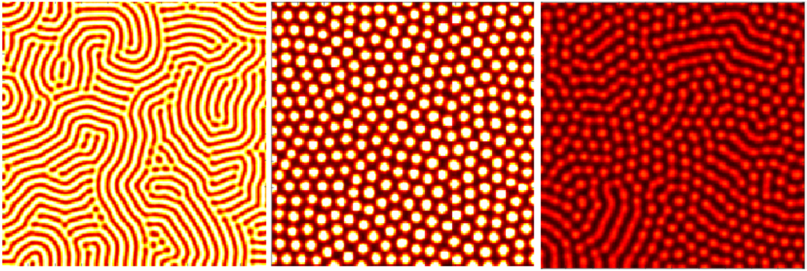
Turing patterns in a static 2D domain



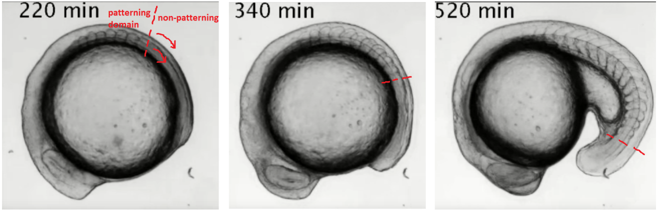
Many stripe patterns in biology are organised...

... and aligned along a preferred direction

What is the mechanism for stripe alignment?

- Specific initial/boundary conditions
- Gradient in reaction parameters
- Anisotropic diffusion
- ...

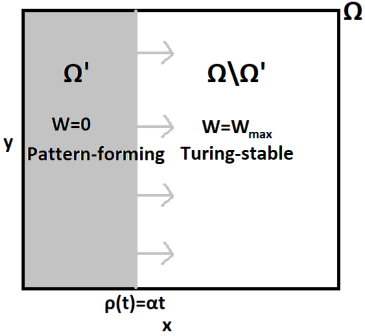
Our hypothesis: Apical domain growth due to “wave of competence” can control stripe organisation



(From video by Andrew Oates)

Reaction-diffusion system with imposed wave of competence

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + f(u,v,W)$$
$$\frac{\partial u}{\partial t} = D_v \nabla^2 v + g(u,v,W)$$
$$W = \begin{cases} 0 & \text{in } \Omega' \\ W_{\text{max}} & \text{in } \Omega \setminus \Omega' \end{cases}$$



The homogeneous steady state is Turing-stable if $W = W_{\text{max}}$ and unstable if $W = 0$.

The reaction terms

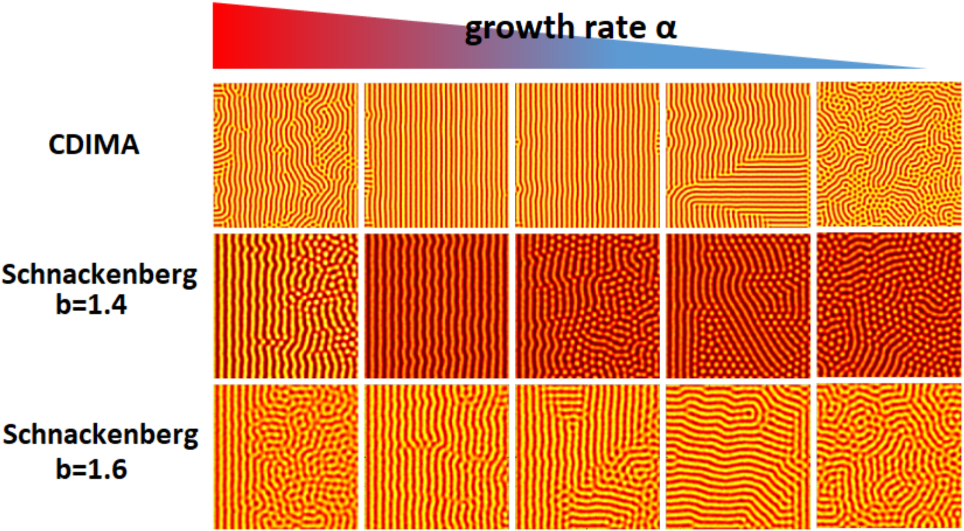
CDIMA (Chlorine Dioxide–Iodine–Malonic Acid) model (Konow et al 2019):

$$f(u,v,W) = a - u - \frac{4uv}{1+u^2} - W$$
$$g(u,v,W) = b\left(u - \frac{uv}{1+u^2} + W\right)$$

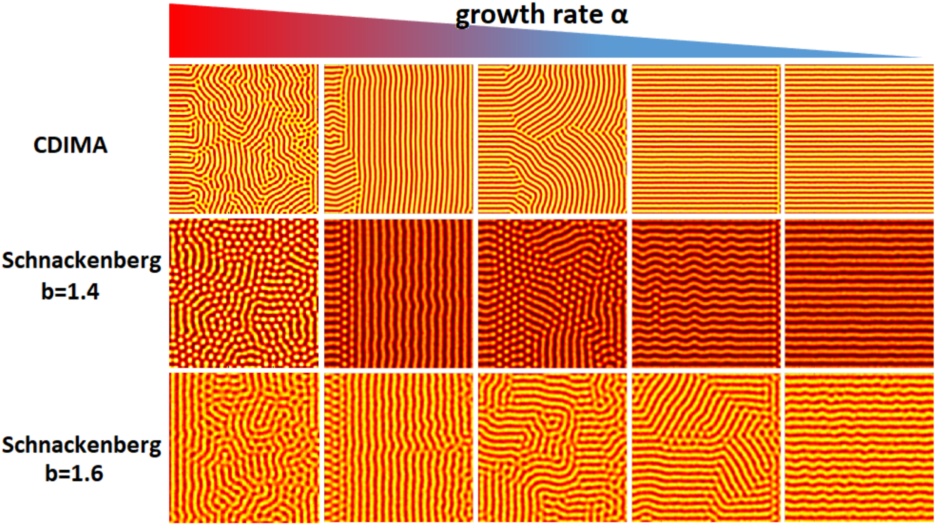
Schnackenberg (1979):

$$f(u,v,W) = a - u + u^2 v + W$$
$$g(u,v,W) = b - u^2 v$$

Numerical simulations: uniform state + noise

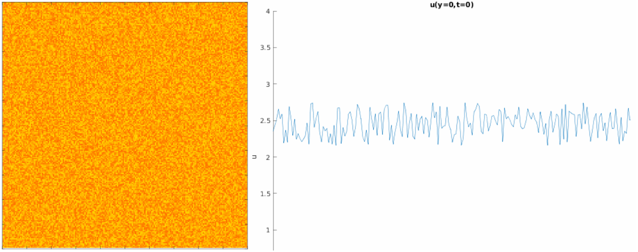


Numerical simulations: horizontal stripes + noise

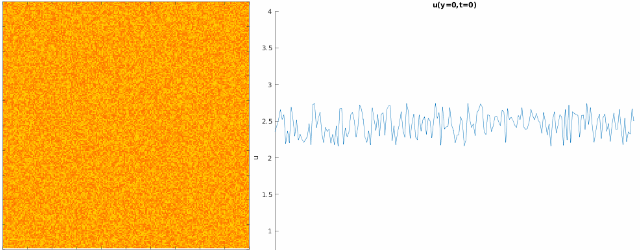


Numerical simulations: narrow 2D domain

Slow domain growth



Fast domain growth



Conclusion: the three modes of stripe patterns

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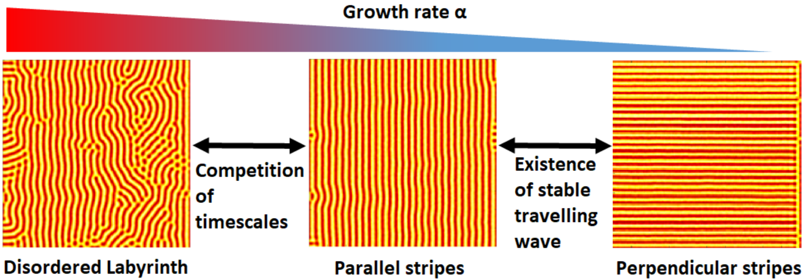

Robustness with respect to noise and kinetic terms
Ongoing Analysis: bifurcation with spectral methods, asymptotics

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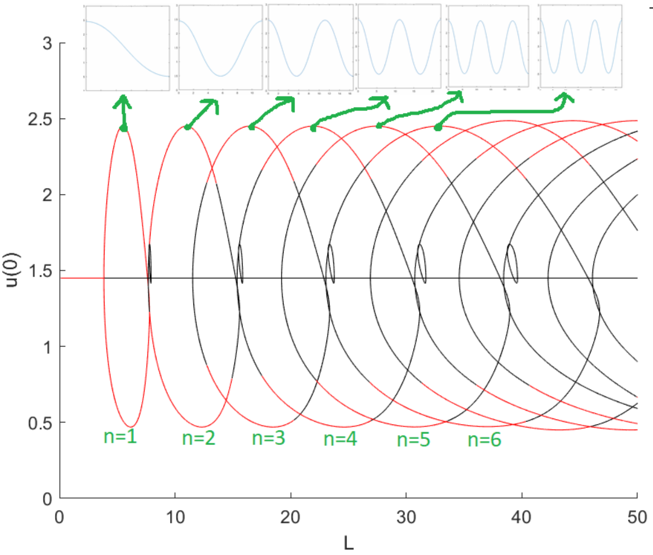
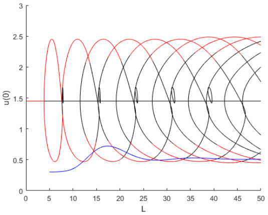
Questions welcome: yue.liu@maths.ox.ac.uk

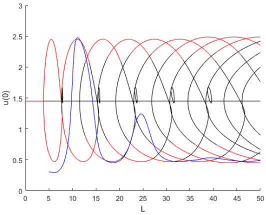
Bifurcation analysis with spectral method + AUTO



We can use this bifurcation diagram to more accurately visualize how does the solution evolve on a 1D expanding domain:



(a) $\alpha = 0.75$



(b) $\alpha = 0.1$

Figure: The trajectory of the PDE solution (blue) combined with the bifurcation diagram. Notice for high growth rates, the solution jump from one branch to the next, while for slow growth rates it follows the branches more closely.